Summer Review Packet for Students Entering Calculus

Name:____________________

Due Date: The last class day of the second week of school.

The purpose of this assignment is to have you practice the mathematical skills necessary to be successful in AP Calculus AB. Most of the skills covered in this packet are skills from Algebra 2 and Pre-Calculus, some are skills from an introduction to the Calculus. If you need to, you may use reference materials to assist you and refresh your memory (old notes, textbooks, online resources, IXL, etc.). While the graphing calculators will be used in class, there are *no calculators allowed* on this packet. You should be able to do everything without a calculator. Optional IXL and videos will also be assigned.

AP Calculus AB is a fast paced course that is taught at the college level. There is a lot of material in the curriculum that must be covered before the AP exam in May. Therefore, we cannot spend a lot of class time re-teaching prerequisite skills. This is why you have this packet. Spend some time with it and make sure you are clear on everything covered in the packet so that you will be successful in Calculus. Of course, you are always encouraged to seek help from your teacher if necessary.

Your assignment is to do at least the odd-numbered problems. Do more if you feel you need the practice. This assignment is worth 50 points.

This assignment will be collected and graded the last class day of the second week of school. Be sure to show all appropriate work to support your answers. In addition, there may be a quiz on this material during the first quarter.

Good Luck!

Mr. Riley

Complex Fractions

When simplifying complex fractions, multiply by a fraction equal to 1, which has a numerator and denominator composed of the common denominator of all the denominators in the complex fraction.

Example:

$$\frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} = \frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} \bullet \frac{x+1}{x+1} = \frac{-7x-7-6}{5} = \frac{-7x-13}{5}$$

$$\frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} = \frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} \bullet \frac{x(x-4)}{x(x-4)} = \frac{-2(x-4)+3x(x)}{5(x)(x-4)-1(x)} = \frac{-2x+8+3x^2}{5x^2-20x-x} = \frac{3x^2-2x+8}{5x^2-21x}$$

Simplify each of the following.

1) $\dfrac{\frac{25}{a}-a}{5+a}$

2) $\dfrac{2-\frac{4}{x+2}}{5+\frac{10}{x+2}}$

3) $\dfrac{4-\frac{12}{2x-3}}{5+\frac{15}{2x-3}}$

4) $\dfrac{\frac{x}{x+1}-\frac{1}{x}}{\frac{x}{x+1}+\frac{1}{x}}$

5) $\dfrac{1-\frac{2x}{3x-4}}{x+\frac{32}{3x-4}}$

Functions

To evaluate a function for a given value, simply plug the value into the function for x.

Recall: $\left(f \circ g\right)(x) = f(g(x))\ OR\ f[g(x)]$ read "***f*** of ***g*** of ***x***" Means to plug the inside function (in this case g(x)) in for x in the outside function (in this case, f(x)).

Example: Given $f(x) = 2x^2 + 1$ *and* $g(x) = x - 4$ find *f(g(x))*.

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

Let $f(x) = 2x + 1$ *and* $g(x) = 2x^2 - 1$**. Find each.**

6) $f(2) =$ ____________

7) $g(-3) =$ ______________

8) $f(a+1) =$ __________

9) $f\left[g(-2)\right] =$__________

10) $g\left[f(m+2)\right] =$___________

11) $\dfrac{f(x+h) - f(x)}{h} =$______

Let $f(x) = \sin x$ **Find each exactly.**

12) $f\left(\dfrac{\pi}{2}\right) =$___________

13) $f\left(\dfrac{2\pi}{3}\right) =$_______________

Let $f(x) = x^2,\ g(x) = 2x + 5,$ *and* $h(x) = x^2 - 1$**. Find each.**

14) $h\left[f(-2)\right] =$ _______

15) $f\left[g(x-1)\right] =$ _______

16) $g\left[h(x^3)\right] =$ _______

Find $\frac{f(x+h)-f(x)}{h}$ for the given function *f*.

17) $f(x) = 9x + 3$

18) $f(x) = 5 - 2x$

Intercepts and Points of Intersection

To find the x-intercepts, let y = 0 in your equation and solve.
To find the y-intercepts, let x = 0 in your equation and solve.

Example: $y = x^2 - 2x - 3$

$\underline{x - \text{int}.\ (Let\ y = 0)}$

$0 = x^2 - 2x - 3$

$0 = (x-3)(x+1)$

$x = -1\ or\ x = 3$

$x - \text{intercepts}\ (-1,0)\ and\ (3,0)$

$\underline{y - \text{int}.\ (Let\ x = 0)}$

$y = 0^2 - 2(0) - 3$

$y = -3$

$y - \text{intercept}\ (0,-3)$

Find the x and y intercepts for each.

19) $y = 2x - 5$

20) $y = x^2 + x - 2$

21) $y = x\sqrt{16 - x^2}$

22) $y^2 = x^3 - 4x$

Use substitution or elimination method to solve the system of equations.

Example:

$$x^2 + y - 16x + 39 = 0$$
$$x^2 - y^2 - 9 = 0$$

Elimination Method

$2x^2 - 16x + 30 = 0$

$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$x = 3 \text{ and } x = 5$

Plug $x = 3$ *and* $x = 5$ into one original

$3^2 - y^2 - 9 = 0 \qquad 5^2 - y^2 - 9 = 0$

$-y^2 = 0 \qquad 16 = y^2$

$y = 0 \qquad y = \pm 4$

Points of Intersection $(5,4), (5,-4)$ *and* $(3,0)$

Substitution Method

Solve one equation for one variable.

$y^2 = -x^2 + 16x - 39$ (1st equation solved for y)

$x^2 - (-x^2 + 16x - 39) - 9 = 0$ Plug what y^2 is equal to into second equation.

$2x^2 - 16x + 30 = 0$ (*The rest is the same as previous example*)

$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$x = 3 \text{ or } x - 5$

Find the point(s) of intersection of the graphs for the given equations.

23) $x + y = 8$
$4x - y = 7$

24) $x^2 + y = 6$
$x + y = 4$

25) $x^2 - 4y^2 - 20x - 64y - 172 = 0$
$16x^2 + 4y^2 - 320x + 64y + 1600 = 0$

Interval Notation

26) Complete the table with the appropriate notation or graph.

Solution	**Interval Notation**	**Graph**
$-2 < x \le 4$		
	$[-1,7)$	
		

Solve each equation. State your answer in BOTH interval notation and graphically.

27) $2x - 1 \ge 0$

28) $-4 \le 2x - 3 < 4$

29) $\frac{x}{2} - \frac{x}{3} > 5$

Domain and Range

Find the domain and range of each function. Write your answer in INTERVAL notation.

30) $f(x) = x^2 - 5$

31) $f(x) = -\sqrt{x + 3}$

32) $f(x) = 3\sin x$

33) $f(x) = \frac{2}{x - 1}$

Inverses

To find the inverse of a function, simply switch the x and the y and solve for the new "y" value.

Example:

$f(x) = \sqrt[3]{x+1}$	Rewrite f(x) as y
$y = \sqrt[3]{x+1}$	Switch x and y
$x = \sqrt[3]{y+1}$	Solve for your new y
$(x)^3 = \left(\sqrt[3]{y+1}\right)^3$	Cube both sides
$x^3 = y+1$	Simplify
$y = x^3 - 1$	Solve for y
$f^{-1}(x) = x^3 - 1$	Rewrite in inverse notation

Find the inverse for each function.

34) $f(x) = 2x+1$

35) $f(x) = \dfrac{x^2}{3}$

Also, recall that to PROVE one function is an inverse of another function, you need to show that: $f(g(x)) = g(f(x)) = x$

Example:

If: $f(x) = \dfrac{x-9}{4}$ *and* $g(x) = 4x+9$ **show *f(x) and g(x)* are inverses of each other.**

$$f(g(x)) = 4\left(\frac{x-9}{4}\right)+9$$
$$= x-9+9$$
$$= x$$

$$g(f(x)) = \frac{(4x+9)-9}{4}$$
$$= \frac{4x+9-9}{4}$$
$$= \frac{4x}{4}$$
$$= x$$

$f(g(x)) = g(f(x)) = x$ therefore they are inverses of each other.

Prove *f and g* are inverses of each other.

36) $f(x) = \frac{x^3}{2}$ $g(x) = \sqrt[3]{2x}$

37) $f(x) = 9 - x^2, x \geq 0$ $g(x) = \sqrt{9 - x}$

Equation of a line

Slope intercept form: $y = mx + b$

Vertical line: x = c (slope is undefined)

Point-slope form: $y - y_1 = m(x - x_1)$

Horizontal line: y = c (slope is 0)

38) Use slope-intercept form to find the equation of the line having a slope of 3 and a y-intercept of 5.

39) Determine the equation of a line passing through the point (5, -3) with an undefined slope.

40) Determine the equation of a line passing through the point (-4, 2) with a slope of 0.

41) Use point-slope form to find the equation of the line passing through the point (0, 5) with a slope of 2/3.

42) Find the equation of a line passing through the point (2, 8) and parallel to the line $y = \frac{5}{6}x - 1$.

43) Find the equation of a line perpendicular to the y- axis passing through the point (4, 7).

44) Find the equation of a line passing through the points (-3, 6) and (1, 2).

45) Find the equation of a line with an x-intercept (2, 0) and a y-intercept (0, 3).

Radian and Degree Measure

Use $\frac{180^\circ}{\pi\, radians}$ to get rid of radians and convert to degrees.

Use $\frac{\pi\, radians}{180^\circ}$ to get rid of degrees and convert to radians.

46) Convert to degrees: a. $\frac{5\pi}{6}$ b. $\frac{4\pi}{5}$ c. 2.63 radians

47) Convert to radians: a. 45° b. -17° c. 237°

Angles in Standard Position

48) Sketch the angle in standard position.

a. $\frac{11\pi}{6}$ b. 230° c. $-\frac{5\pi}{3}$ d. 1.8 radians

Reference Triangles

49) Sketch the angle in standard position. Draw the reference triangle and label the sides, if possible.

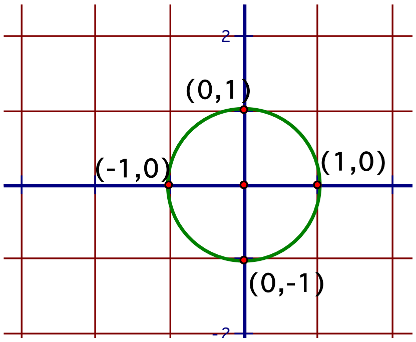
a. $\frac{2}{3}\pi$ b. 225°

c. $-\frac{\pi}{4}$ d. 30°

Unit Circle

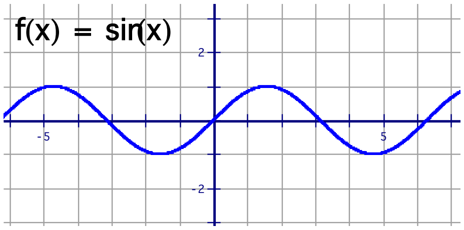
You can determine the sine or cosine of a quadrantal angle by using the unit circle. The x-coordinate of the circle is the cosine and the y-coordinate is the sine of the angle.

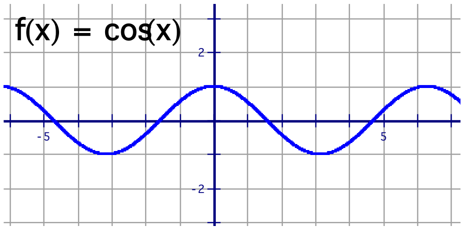
Example: $\sin 90^\circ = 1$ $\qquad \cos\frac{\pi}{2} = 0$



50) $a.)\ \sin 180^\circ$ $\quad b.)\ \cos 270^\circ$ $\quad c.)\ \sin(-90^\circ)$ $\quad d.)\ \sin\pi$ $\quad e.)\ \cos 360^\circ$ $\quad f.)\ \cos(-\pi)$

Graphing Trig Functions





y = sin x and y = cos x have a period of 2 π and an amplitude of 1. Use the parent graphs above to help you sketch a graph of the functions below. For $f(x) = A\sin(Bx + C) + K$, A = amplitude, $\frac{2\pi}{B}$ = period, $\frac{C}{B}$ = phase shift (positive C/B shift left, negative C/B shift right) and K = vertical shift.

Graph two complete periods of the function.

51) $f(x) = 5\sin x$

52) $f(x) = \sin 2x$

53) $f(x) = -\cos\left(x - \frac{\pi}{4}\right)$

54) $f(x) = \cos x - 3$

Trigonometric Equations:

Solve each of the equations for $0 \le x < 2\pi$. Isolate the variable, sketch a reference triangle, find all the solutions within the given domain, $0 \le x < 2\pi$. Remember to double the domain when solving for a double angle. Use trig identities, if needed, to rewrite the trig functions. (See formula sheet at the end of the packet.)

55) $\sin x = -\frac{1}{2}$

56) $2\cos x = \sqrt{3}$

57) $\cos 2x = \frac{1}{\sqrt{2}}$

58) $\sin^2 x = \frac{1}{2}$

59) $\sin 2x = -\frac{\sqrt{3}}{2}$

60) $2\cos^2 x - 1 - \cos x = 0$

61) $4\cos^2 x - 3 = 0$

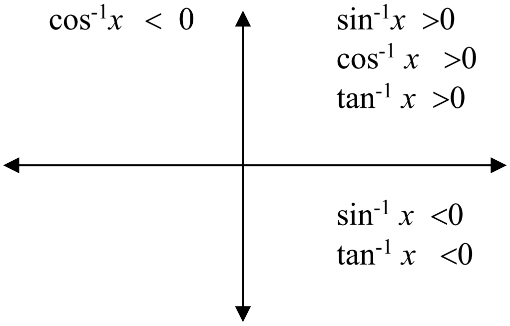
62) $\sin^2 x + \cos 2x - \cos x = 0$

Inverse Trigonometric Functions:

Recall: Inverse Trig Functions can be written in one of ways:

$$\arcsin(x) \qquad\qquad \sin^{-1}(x)$$

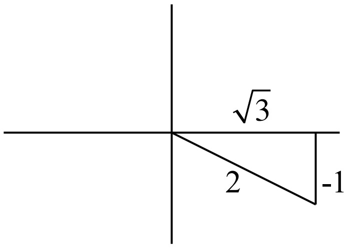
Inverse trig functions are defined only in the quadrants as indicated below due to their restricted domains.



Example:

Express the value of "y" in radians.

$y = \arctan\dfrac{-1}{\sqrt{3}}$ Draw a reference triangle.



This means the reference angle is 30° or $\dfrac{\pi}{6}$. So, y = $-\dfrac{\pi}{6}$ so that it falls in the interval from

$$\frac{-\pi}{2} < y < \frac{\pi}{2} \qquad\qquad \text{Answer: } y = -\frac{\pi}{6}$$

For each of the following, express the value for "y" in radians.

76) $y = \arcsin\dfrac{-\sqrt{3}}{2}$

77) $y = \arccos(-1)$

78) $y = \arctan(-1)$

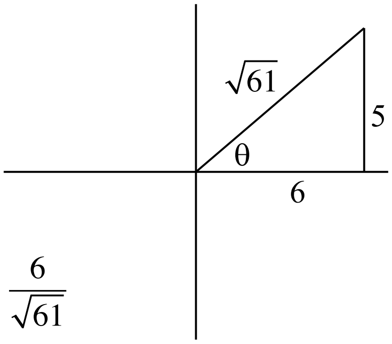
Example: Find the value without a calculator.

$$\cos\left(\arctan\frac{5}{6}\right)$$

Draw the reference triangle in the correct quadrant first.

Find the missing side using Pythagorean Thm.

Find the ratio of the cosine of the reference triangle. $\cos\theta = \frac{6}{\sqrt{61}}$



For each of the following give the value without a calculator.

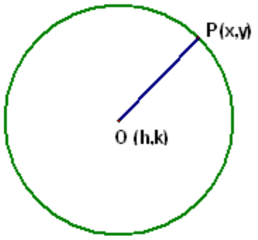
63) $\tan\left(\arccos\frac{2}{3}\right)$

64) $\sec\left(\sin^{-1}\frac{12}{13}\right)$

65) $\sin\left(\arctan\frac{12}{5}\right)$

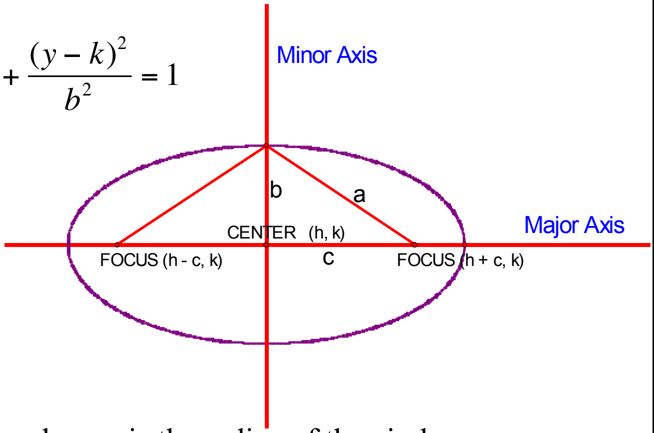
66) $\sin\left(\sin^{-1}\frac{7}{8}\right)$

Circles and Ellipses



$$r^2 = (x-h)^2 + (y-k)^2$$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

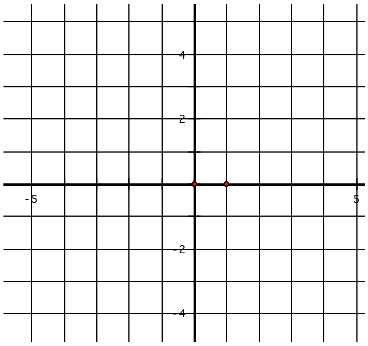


For a circle centered at the origin, the equation is $x^2 + y^2 = r^2$, where **r** is the radius of the circle.

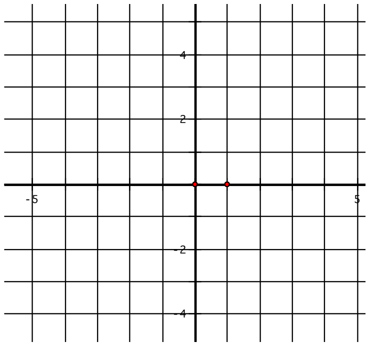
For an ellipse centered at the origin, the equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where **a** is the distance from the center to the ellipse along the x-axis and **b** is the distance from the center to the ellipse along the y-axis. If the larger number is under the y^2 term, the ellipse is elongated along the *y*-axis. For our purposes in Calculus, you will not need to locate the foci.

Graph the circles and ellipses below:

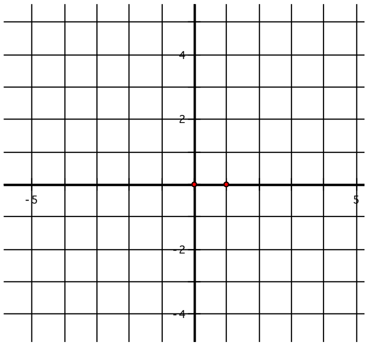
67) $x^2 + y^2 = 16$



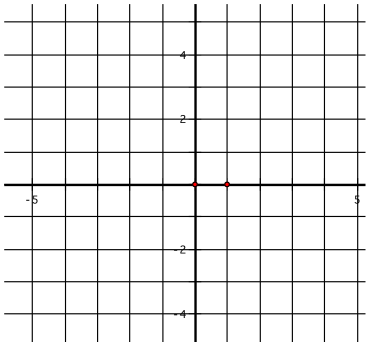
68) $x^2 + y^2 = 5$



69) $\dfrac{x^2}{1} + \dfrac{y^2}{9} = 1$



70) $\dfrac{x^2}{16} + \dfrac{y^2}{4} = 1$



Limits

Finding limits numerically.

Complete the table and use the result to estimate the limit.

71) $\displaystyle\lim_{x\to 4} \frac{x-4}{x^2-3x-4}$

x	3.9	3.99	3.999	4.001	4.01	4.1
f(x)						

72) $\displaystyle\lim_{x\to -5} \frac{\sqrt{4-x}-3}{x+5}$

x	-5.1	-5.01	-5.001	-4.999	-4.99	-4.9
f(x)						

Finding limits graphically.

Find each limit graphically. Use your calculator to assist in graphing.

73) $\displaystyle\lim_{x\to 0} \cos x$

74) $\displaystyle\lim_{x\to 5} \frac{2}{x-5}$

75) $\displaystyle\lim_{x\to 1} f(x)$

$$f(x) = \begin{cases} x^2 + 3, & x \neq 1 \\ 2, & x = 1 \end{cases}$$

Evaluating Limits Analytically

Solve by direct substitution whenever possible. If needed, rearrange the expression so that you can do direct substitution.

76) $\lim_{x \to 2}(4x^2 + 3)$

77) $\lim_{x \to 1} \frac{x^2 + x + 2}{x + 1}$

78) $\lim_{x \to 0} \sqrt{x^2 + 4}$

79) $\lim_{x \to \pi} \cos x$

80) $\lim_{x \to 1}\left(\frac{x^2 - 1}{x - 1}\right)$ HINT: Factor and simplify.

81) $\lim_{x \to -3} \frac{x^2 + x - 6}{x + 3}$

82) $\lim_{x \to 0} \frac{\sqrt{x + 1} - 1}{x}$ HINT: Rationalize the numerator.

83) $\lim_{x \to 3} \frac{3 - x}{x^2 - 9}$

84) $\lim_{h \to 0} \frac{2(x + h) - 2x}{h}$

One-Sided Limits

Find the limit if it exists. First, try to solve for the overall limit. If an overall limit exists, then the one-sided limit will be the same as the overall limit. If not, use the graph and/or a table of values to evaluate one-sided limits.

85) $\lim_{x \to 5^+} \frac{x - 5}{x^2 - 25}$

86) $\lim_{x \to -3^-} \frac{x}{\sqrt{x^2 - 9}}$

87) $\lim_{x\to 10^+} \frac{|x-10|}{x\ -10}$

88) $\lim_{x\to 5^-} \left(-\frac{3}{x+5}\right)$

Vertical Asymptotes

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. That will be the vertical asymptote.

89) $f(x)=\frac{1}{x^2}$

90) $f(x)=\frac{x^2}{x^2-4}$

91) $f(x)=\frac{2+x}{x^2(1-x)}$

Horizontal Asymptotes

Determine the horizontal asymptotes using the three cases below.

Case I. Degree of the numerator is less than the degree of the denominator. The asymptote is y = 0.

Case II. Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Case III. Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

Determine all Horizontal Asymptotes.

92) $f(x)=\frac{x^2-2x+1}{x^3+x-7}$

93) $f(x)=\frac{5x^3-2x^2+8}{4x-3x^3+5}$

94) $f(x)=\frac{4x^5}{x^2-7}$

Determine each limit as x goes to infinity.

RECALL: This is the same process you used to find Horizontal Asymptotes for a rational function.

**** In a nutshell**

1. Find the highest power of x.
2. How many of that type of x do you have in the numerator?
3. How many of that type of x do you have in the denominator?
4. That ratio is your limit!

95. $\lim_{x\to\infty}\left(\frac{2x-5+4x^2}{3-5x+x^2}\right)$

96. $\lim_{x\to\infty}\left(\frac{2x-5}{3-5x+3x^2}\right)$

97. $\lim_{x\to\infty}\left(\frac{7x+6-2x^3}{3+14x+x^2}\right)$

Limits to Infinity

A rational function does not have a limit if it goes to ± ∞, however, you can state the direction the limit is headed if both the left and right hand side go in the same direction.

Determine each limit if it exists. If the limit approaches ∞ *or* $-\infty$, please state which one the limit approaches.

98. $\lim_{x\to -1^+}\frac{1}{x+1}=$

99. $\lim_{x\to 1^+}\frac{2+x}{1-x}=$

100. $\lim_{x\to 0}\frac{2}{\sin x}$

Formula Sheet

Reciprocal Identities: $\csc x=\frac{1}{\sin x}$ $\sec x=\frac{1}{\cos x}$ $\cot x=\frac{1}{\tan x}$

Quotient Identities: $\tan x=\frac{\sin x}{\cos x}$ $\cot x=\frac{\cos x}{\sin x}$

Pythagorean Identities: $\sin^2 x+\cos^2 x=1$ $\tan^2 x+1=\sec^2 x$ $1+\cot^2 x=\csc^2 x$

Double Angle Identities: $\sin 2x=2\sin x\cos x$ $\tan 2x=\frac{2\tan x}{1-\tan^2 x}$

$$\cos 2x=\cos^2 x-\sin^2 x$$
$$=1-2\sin^2 x$$
$$=2\cos^2 x-1$$

Logarithms: $y=\log_a x$ is equivalent to $x=a^y$

Product property: $\log_b mn=\log_b m+\log_b n$ Power property: $\log_b m^p=p\log_b m$

Quotient property: $\log_b\frac{m}{n}=\log_b m-\log_b n$ Property of equality: If $\log_b m=\log_b n$, then m = n

Change of base formula: $\log_a n=\frac{\log_b n}{\log_b a}$

Derivative of a Function: Slope of a tangent line to a curve or the derivative: $\lim_{h\to\infty}\frac{f(x+h)-f(x)}{h}$

Slope-intercept form: $y=mx+b$ Point-slope form: $y-y_1=m(x-x_1)$ Standard form: Ax + By + C = 0